Grade Level/Course:
Algebra 2 and Pre-Calculus

Lesson/Unit Plan Name:
Introduction to Logarithms: A Function Approach

Rationale/Lesson Abstract:
This lesson is designed to introduce logarithms to students by building on their prior knowledge of exponents and properties of inverse functions. It is intended to help students build an understanding of logarithms as the inverse of an exponent and to make the entire concept less abstract and intimidating. Students should have a firm grasp of rational exponents, graphing exponential functions and properties of inverse functions before beginning this lesson.

Timeframe:
Two periods or One block

Common Core Standard(s):

F.IF.1: Interpreting Functions
Understand the concept of a function and use function notation.
Understand that a function from one set (called the domain) to another set (called the range) assigns each element of the domain to exactly one element of the range.

F.IF.7e
Analyze functions using different representations.
Graph exponential and logarithmic functions, showing intercept and end behavior.

F.IF.9 Building Functions
Analyze functions using different representations.
Compare the properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal description).

F.BF.3.1
Solve problems using functional concepts, such as composition defining the inverse function and performing arithmetic operations on functions.

F.BF.4b
Read values of an inverse function from a graph or a table.

F.BF.5
Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Warm-Up

CST/CCSS: 12.0/A.SEE.3

Select *all* of the following that are equivalent to:

$$125^{\frac{4}{3}}$$

A) $(\sqrt[3]{125})^4$

B) $(\sqrt[4]{125})^3$

C) $\sqrt[3]{(125)^4}$

D) $\sqrt[4]{(125)^3}$

E) $(5)^4$

F) $(5)^3$

G) 625

Review: Algebra 2

If $2^x = 64,\ 4^y = 64\ and\ 8^z = 64$, then the values of x, y, and z are:

A) $x = 32,\ y = 16,\ z = 8$

B) $x = 6,\ y = 3,\ z = 2$

C) $x = 6,\ y = 3,\ z = 8$

D) $x = 32,\ y = 3,\ z = 2$

- What is interesting about each of these equations?
- Can you think of any other instances where this may occur concerning exponents?

Current: Algebra 2

Which of the following statements is true about the two functions?

$f(x) = \sqrt{x + 25}$ $\qquad g(x) = x^2 - 25$ for $x \geq 0$

A) The functions are opposites.

B) The functions are both quadratic.

C) The graphs of both functions are lines.

D) The functions are inverses.

- Justify your answer choice graphically, algebraically or using a table.

Other: Grade 7

Select *all* of the following that are equivalent to:

$$10^{-2}$$

A) $\frac{1}{10}$

B) 0.01

C) 0.1

D) $\frac{1}{100}$

E) 0.001

F) $\frac{1}{10^2}$

Instructional Resources/Materials:
Graph paper, student note-taking guide, warm-ups, exponent reference sheet, paper and pencils.

Activity/Lesson

Warm-Up Solutions

$125^{\frac{4}{3}}$ $= \sqrt[3]{(125)^4}$ $= (\sqrt[3]{125})^4$ $= (5)^4$ $= 625$	Answers A, C, E and G are all equivalent to the original problem. Use this problem to review fractional exponents.	$2^x = 64 \quad 2^6 = 64$ $4^y = 64 \quad 4^3 = 64$ $8^z = 64 \quad 8^2 = 64$ *Each of the equations is equal to 64 but each power has a different base and exponent. *This also happens with 3^4 and 9^2.
*The functions *f* and *g* are inverses. We know this because the composition of the two functions is equal to *x*. Also, show graph with reflection in line *y*=*x*. Mention any other ways to show that two functions are inverses.	$f \circ g$ $= f(g(x))$ $= \sqrt{(x^2 - 25) + 25}$ $= \sqrt{x^2 - 25 + 25}$ $= \sqrt{x^2 + 0}$ $= \sqrt{x^2}$ $= x$	The expression 10^{-2} is equivalent to: B) 0.01 D) $\frac{1}{100}$ F) $\frac{1}{10^2}$ Use this problem to review negative exponents and converting fractions to decimals.

Introduction to Logarithms

Graphing Activity: Part 1
Graph the exponential function $g(x) = 2^x$. (See page 4) Be sure to remind students that the variable in this function is found in the exponent. Use a table to show all work. Also, graph the line $h(x) = x$ on the same plane. Save for later.

Activity: Play "Guess my exponent." Use this activity to help introduce solving exponential equations in an informal way. Use an exponent reference sheet as students need help.

Guess My Exponent (Solutions)

$10^x = 1000$ $x = 3$	$5^b = 625$ $b = 4$	$3^z = 81$ $z = 4$	$\left(\frac{1}{81}\right)^t = 9$ $t = -\frac{1}{2}$
$10^y = \frac{1}{100}$ $y = -2$	$125^a = 5$ $a = \frac{1}{3}$	$7^d = 343$ $d = 3$	$16^w = 64$ $w = \frac{3}{2}$

Introduction to the Definition of a Logarithm: Go through each example, asking students to pay attention to each of the equivalent forms. Show the definition of a logarithm without actually telling students the pattern; ask them to make observations and predictions to help them understand and state the definition. Also, point out that each exponential form is <u>true.</u>

Exponential Form	**Logarithmic Form**
$2^5 = 32$	$\log_2 32 = 5$
$3^4 = 81$	$\log_3 81 = 4$
$4^{-2} = \frac{1}{16}$	$\log_4 \frac{1}{16} = -2$
$5^3 = 125$	$\log_5 125 = 3$
$10^{-2} = \frac{1}{100}$	$\log \frac{1}{100} = -2$ (The Common Logarithm has base 10)
$81^{\frac{1}{2}} = 9$	$\log_{81} 9 = \frac{1}{2}$

Once students have established the pattern, write the following on the board:

$$\log_{base}(result) = exponent \qquad base^{exponent} = result$$

Give the formal definition of a logarithm.

Definition: The **logarithm** of a number is the exponent by which another fixed value, the **base**, must be raised to produce that number. In general, if $x = b^y$, then $\log_b x = y$.

Use this definition to complete the following **examples**:

Example 1:	**Example 2:**	**Example 3:**	**Example 4:**	**Example 5:**
$\log_2 16 = x$ $2^x = 16$ $2^x = 2^4$ $x = 4$	$\log_{12} y = -2$ $12^{-2} = y$ $\left(\frac{1}{12}\right)^2 = y$ $\frac{1}{144} = y$ $y = \frac{1}{144}$	$\log_a 216 = 3$ $a^3 = 216$ $a^3 = 6^3$ $a = 6$	$\log_2 \frac{1}{32} = b$ $2^b = \frac{1}{32}$ $2^b = \frac{1}{2^5}$ $2^b = 2^{-5}$ $b = -5$	$\log_2 2 = z$ $2^z = 2$ $2^z = 2^1$ $z = 1$

Graphing Activity: Part 2

Graph the logarithmic function $f(x) = \log_2 x$ on the same plane as Part 1. (See page 4) Be sure to remind students that the variable in this function is part of a logarithm and that they should use the definition of a logarithm in order to find solutions; they should convert to exponential form. Use a table to show all work.

Verifying Inverse Functions:

After students have completed the table and graphed, ask them to examine the graphs of the two functions and the table of values associated with each. They should be able to say that the functions are inverses for two reasons: the graphs of the functions are symmetric about the line $h(x) = x$ and that the tables of values for each function have a domain and range that are reversed. Further, ask them if they can think of another way to show this; use leading questions to help them to consider finding the composition of the two functions or, $f \circ g(x)$. Remind them that if the result of doing this composition is *x*, that the two functions are indeed inverses.

- **Verify algebraically** that the functions are inverses.

(**Note:** The function may be composed as $f \circ g(x)$ or as $g \circ f(x)$; if the two are inverses, the result of either composition will be *x*. However, each of these requires additional information, therefore, we will only perform the composition $f \circ g(x)$. The other may be revisited after more information on logarithms is given.)

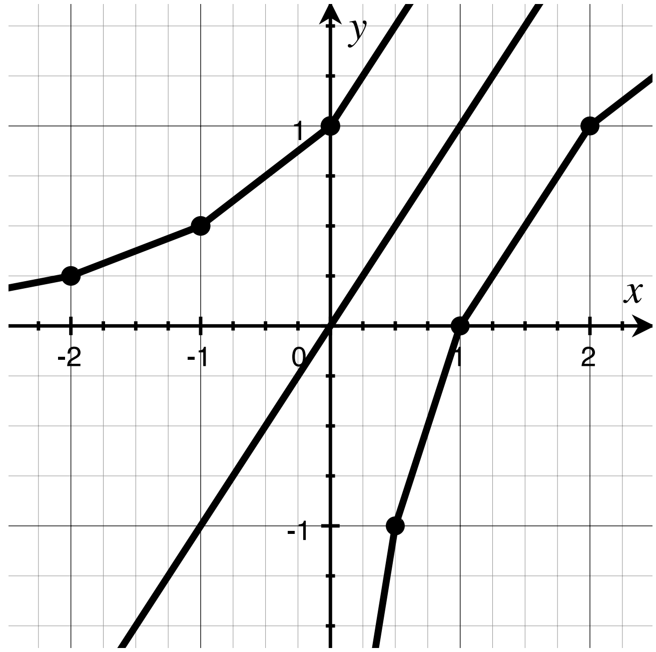
Inform students that there is a Law of Logarithms that allows the exponent contained in a logarithm to be moved to the front of the log expression as a factor. *Remind students that we may set up the following: $\log_2 2 = w$ $2^w = 2^1$ $w = 1$ $\log_2 2 = 1$ Therefore, we may substitute 1 in place of $\log_2 2$.	$f \circ g(x)$ $= f(g(x))$ $= \log_2(2^x)$ $= \log_2 2^x$ $= x \bullet \log_2 2$ $= x \bullet 1$ $= x$	$g \circ f(x)$ $= g(f(x))$ $= g(\log_2 x)$ $= 2^{\log_2 x}$ Let $y = 2^{\log_2 x}$ Using the pattern, $\log_2 y = \log_2 x$ $\therefore y = x$

Graphing Activity

Part 1: Graph the line $h(x) = x$.

Part 2: Graph the function $g(x) = 2^x$.
Please use the table provided and show all work.

Part 3: Graph the function $f(x) = \log_2 x$.
Please use the table provided and show all work.



x	$f(x) = \log_2 x$	y	(x, y)
$\frac{1}{4}$	$y = \log_2 \frac{1}{4}$ $2^y = \frac{1}{4}$ $y = -2$	-2	$\left(\frac{1}{4}, -2\right)$
$\frac{1}{2}$	$y = \log_2 \frac{1}{2}$ $2^y = \frac{1}{2}$ $y = -1$	-1	$\left(\frac{1}{2}, -1\right)$
1	$y = \log_2 1$ $2^y = 1$ $y = 0$	0	(1, 0)
2	$y = \log_2 2$ $2^y = 2$ $y = 1$	1	(2, 1)
4	$y = \log_2 4$ $2^y = 4$ $y = 2$	2	(4, 2)

x	$g(x) = 2^x$	y	(x, y)
-2	$y = 2^{-2}$ $y = \frac{1}{4}$	$\frac{1}{4}$	$\left(-2, \frac{1}{4}\right)$
-1	$y = 2^{-1}$ $y = \frac{1}{2}$	$\frac{1}{2}$	$\left(-1, \frac{1}{2}\right)$
0	$y = 2^0$ $y = 1$	1	(0, 1)
1	$y = 2^1$ $y = 2$	2	(1, 2)
2	$y = 2^2$ $y = 4$	4	(2, 4)

Introduction to Logarithms: Note-Taking Guide **Name: ____________________________**

Guess My Exponent

$10^x = 1000$ $x =$	$5^b = 625$ $b =$	$3^z = 81$ $z =$	$\left(\frac{1}{81}\right)^t = 9$ $t =$
$10^y = \frac{1}{100}$ $y =$	$125^a = 5$ $a =$	$7^d = 343$ $d =$	$16^w = 64$ $w =$

Exponential Form	Logarithmic Form

Did you notice any patterns?
Can you create a rule?

Definition of a Logarithm:

Practice Problems:

Example 1:	Example 2:	Example 3:	Example 4:	Example 5:

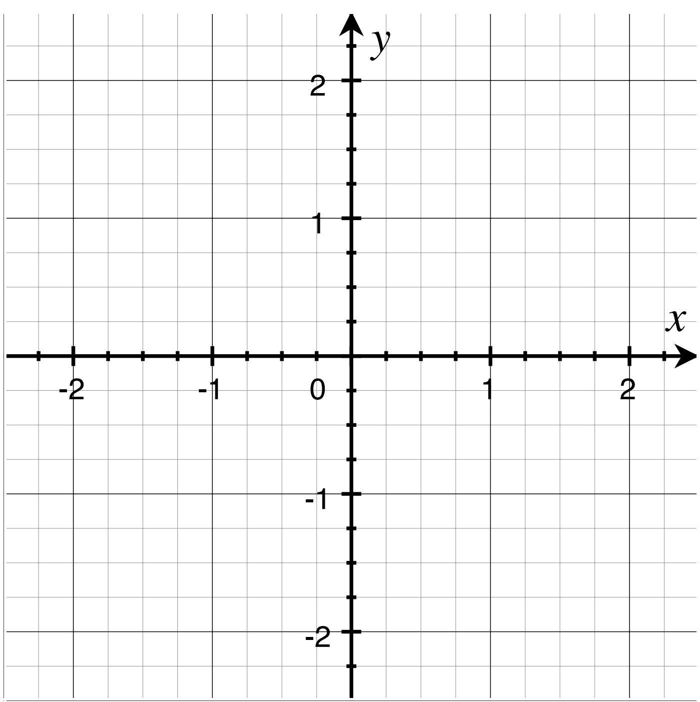
Graphing Activity

Part 1: Graph the line $h(x) = x$.

Part 2: Graph the function $g(x) = 2^x$.
Please use the table provided and show all work.

Part 3: Graph the function $f(x) = \log_2 x$.
Please use the table provided and show all work.

x	$g(x) = 2^x$	y	(x, y)
-2			
-1			
0			
1			
2			



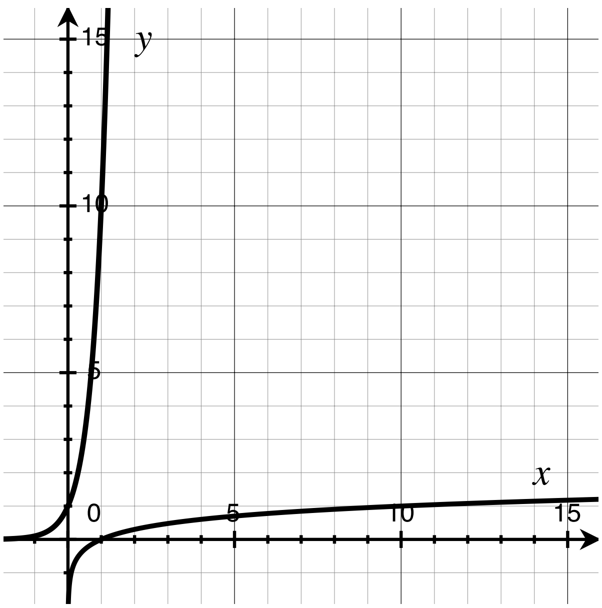
x	$f(x) = \log_2 x$	y	(x, y)
$\frac{1}{4}$			
$\frac{1}{2}$			
1			
2			
4			

Exit Ticket: Constructed Response

Jason has been asked to show that the two functions shown below are inverses.

$f(x) = \log x$ $\qquad\qquad$ $g(x) = 10^x$

a. Jason decided to graph each function. The graphs are shown below.
How does his graph show that the functions *are* inverses?



b. Show algebraically that the functions are inverses. In one to three sentences, explain how your work demonstrates that they are inverses.

Sample Top-Score Responses

Part a:

Method: Examine ordered pairs for each function

Show the following tables for each function:

$f(x) = \log x$

x	y
1	0
10	1

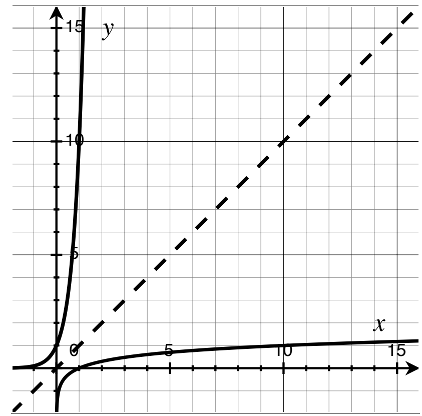
$g(x) = 10^x$

x	y
0	1
1	10

The domain and range for each of the functions are interchanged, which, by definition, shows that the two functions are inverses.

Method: Show symmetry about $y = x$

Graph the line $y = x$ on the given graph.



The graphs show symmetry about the line $y = x$, therefore, by definition, the two functions are inverses.

Part b:

In order to show algebraically that two functions are inverses, we may find the inverse of $f(x) = \log x$ and show that its inverse is $g(x) = 10^x$, or we may show that the inverse of $g(x) = 10^x$ is $f(x) = \log x$. We may also compose the two functions and show that the composition of $f(g(x)) = x$ or $g(f(x)) = x$.

Method: Find inverse functions

Find the inverse function of $f(x) = \log x$.

$$f(x) = \log x$$
$$y = \log x$$
$$y = \log_{10} x$$
$$x = \log_{10} y$$
$$10^x = y$$
$$f^{-1}(x) = 10^x$$

Since $f^{-1}(x) = g(x) = 10^x$, the functions are inverses.

Find the inverse function of $g(x) = 10^x$.

$$g(x) = 10^x$$
$$y = 10^x$$
$$x = 10^y$$
$$\log x = \log 10^y$$
$$\log_{10} x = \log_{10} 10^y$$
$$\log_{10} x = y \bullet \log_{10} 10$$
$$\log_{10} x = y \bullet 1$$
$$\log x = y$$
$$g^{-1}(x) = \log x$$

Since $g^{-1}(x) = f(x) = \log x$, the functions are inverses.

Method: Function composition

Find the composition of the functions both ways.

$$f(g(x)) = \log_{10} 10^x$$
$$f(g(x)) = x \bullet \log_{10} 10$$
$$f(g(x)) = x \bullet 1$$
$$f(g(x)) = x$$

$$g(f(x)) = 10^{\log_{10} x}$$
$$g(f(x)) = x$$

Since the composition of $f(g(x)) = g(f(x)) = x$, the functions are inverses.

Common Core Standards

Building Functions	F.BF

Analyze functions using different representations.

9. Compare the properties of two functions each represented in a different way. (Algebraically, graphically, numerically in tables, or by verbal description)

Scoring Rubric:

3 points:

The student creates a table of values showing that the domain and range are reversed and/or graphs the line $y = x$ on the graph and uses this to show symmetry and thus, to justify why the two functions are inverses. The student explains his or her thinking using complete sentences, including stating that the domain and range are reversed and that the graphs of the two functions are symmetric about the line $y = x$. The student verifies algebraically that the two functions are inverses by either finding inverse functions or by performing function composition. The student also explains, using complete sentences, that the inverses are equal to the other, given function or that the function composition is equal to x, and that, by properties of inverse functions, $f(x)$ and $g(x)$ are inverse functions.

2 points:

The student may write ordered pairs from each function and point out that the x and y values are reversed but may not indicate why this is significant. The student may graph the line $y = x$ to show symmetry but may not completely or accurately explain the significance of the symmetry. The student may attempt to solve for the inverse functions or compose the two functions but does so with minor mathematical errors. The student may attempt to explain, in writing, why he or she is doing this but the explanation may not be complete or entirely accurate.

1 point:

The student may attempt to write down information from the graph but does not offer any explanation as to why it would be significant. The student may try to solve for the inverse functions or compose the two functions but does so with no written explanation and with many mathematical errors.

0 points:

There is little or no evidence of student work, written or mathematical. The problem is left blank.